education

Department:
Education
PROVINCE OF KWAZULU-NATAL

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2020

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of **10** questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

The total number of red cards issued per country to players during a soccer competition are given in the table below:

NUMBER OF RED CARDS	NUMBER OF COUNTRIES (f)	MIDPOINT OF INTERVAL (x)	$f.x$
$0 < x \leq 2$	27		
$2 < x \leq 4$	15		
$4 < x \leq 6$	5		
$6 < x \leq 8$	5		
$8 < x \leq 10$	3		
TOTAL			

1.1 Calculate the estimated mean of the number of red cards per country. (3)

1.2 Draw an ogive curve to represent the above data. (3)

1.3 Calculate the interquartile range of the number of red cards issued per country in the competition. (2)

[8]

QUESTION 2

The table below shows a relationship between the monthly rent (x) a person pays for an apartment and the person's monthly income (y). Both are given in thousands of rands.

YEAR	2003	2004	2005	2006	2007	2008
Rent (x)	2	3	3,5	5,2	5,6	6
Income (y)	9	13,5	15	16,5	17	20

2.1 Determine the equation of the regression line. (4)

2.2 Determine the estimated monthly income if the rent per month is R9000. (2)

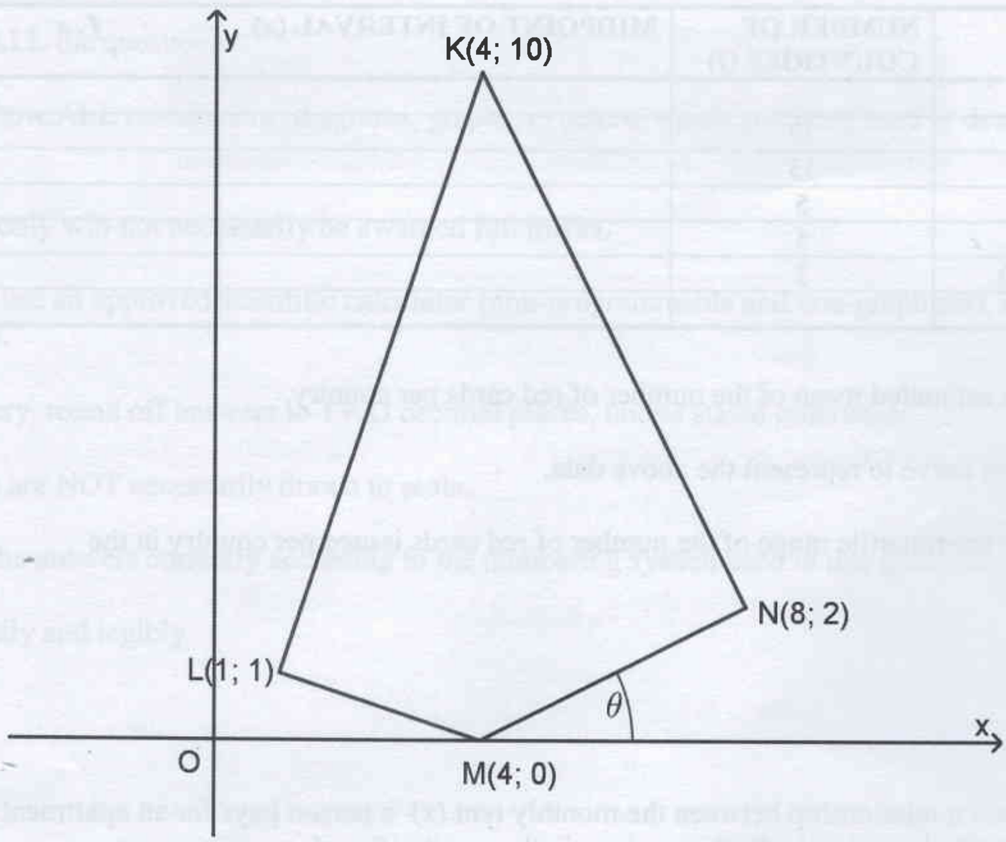
2.3 Calculate the value of the correlation coefficient. (2)

2.4 Describe the relationship between the monthly rent and the monthly income. (2)

[10]

QUESTION 3

In the diagram KLMN is a quadrilateral with K(4; 10), L(1; 1), M(4; 0) and N(8; 2).



3.1 Determine the:

3.1.1 gradient of LM and MN (4)

3.1.2 length of KM. (2)

3.1.3 value of θ (2)

3.1.4 midpoint of LN (2)

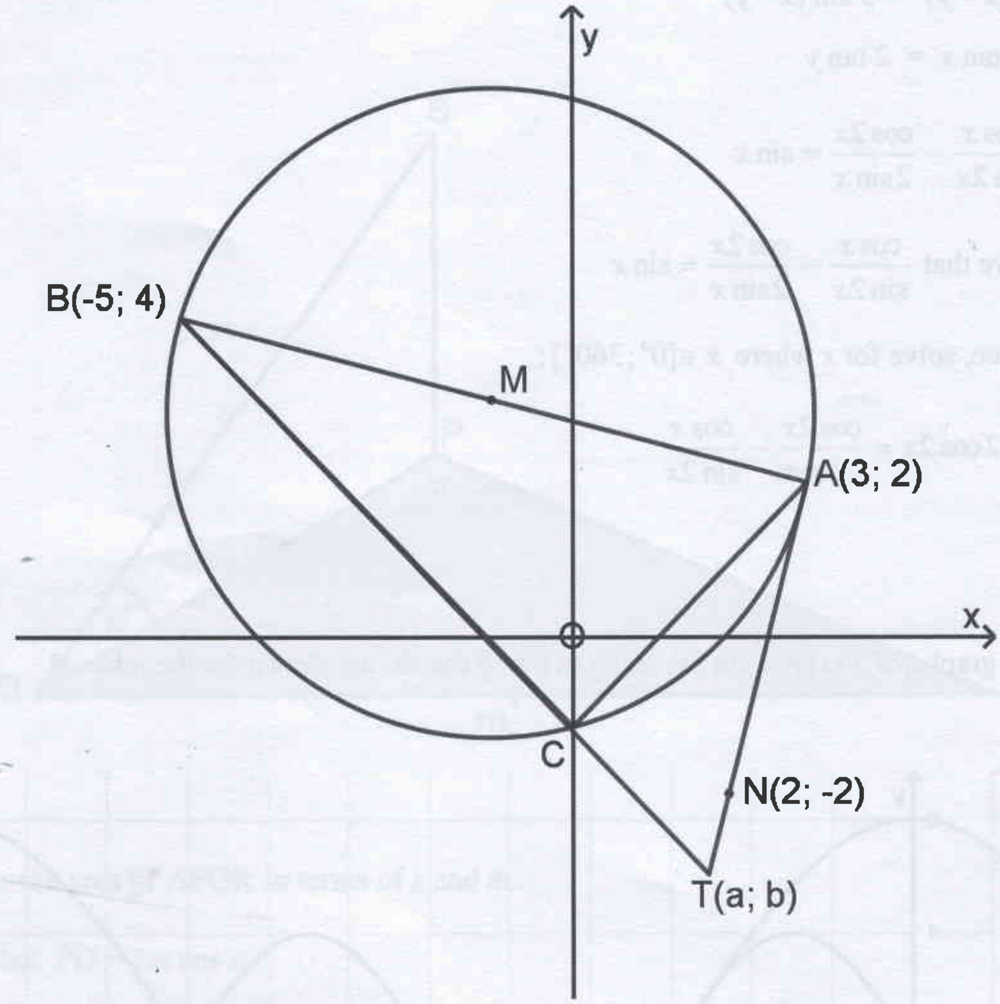
3.2 Show that KL $\perp$ LM (3)

3.3 Prove that KLMN is a cyclic quadrilateral. (4)

[17]

QUESTION 4

In the sketch below, AB is a diameter with coordinates A(3; 2) and B(–5; 4) of circle ABC. M is the centre of the circle. BC produced meets AT in T. N(2; –2) is a point on the line TA. C is the y – intercept of the circle.



4.1 Determine the co-ordinates of M the centre of the circle (2)

4.2 Write down the equation of the circle in the form $(x-p)^2 + (y-q)^2 = r^2$ (3)

4.3 Prove that TA is a tangent to the circle at A. (5)

4.4 Determine the equations of the lines

4.4.1 TA and (4)

4.4.2 BT (6)

4.5 If the coordinates of T are $(a; b)$, calculate the values of a and b. (3)

[23]

QUESTION 5

5.1 Without using a calculator, evaluate

$\text{Cos } 79^\circ \cos 311^\circ + \sin 101^\circ \sin 49^\circ$ (4)

5.2 Given: $\sin(x + y) = 3\sin(x - y)$

Prove that: $\tan x = 2\tan y$ (4)

5.3 Given: $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$

5.3.1 Prove that $\frac{\cos x}{\sin 2x} - \frac{\cos 2x}{2\sin x} = \sin x$ (4)

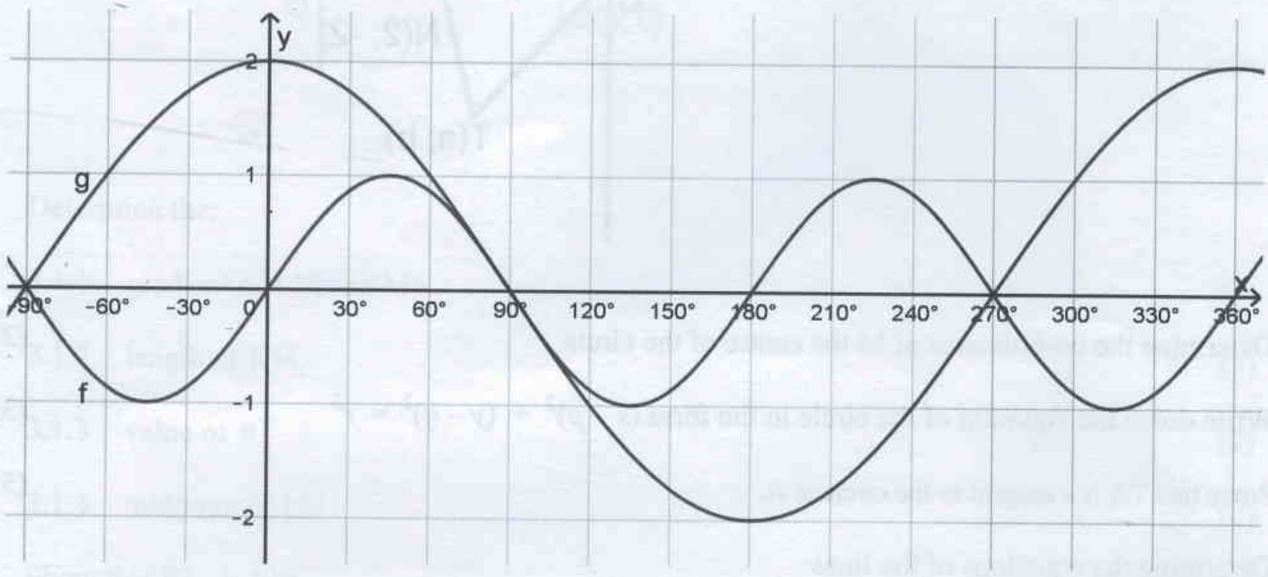
5.3.2 Hence, solve for x where $x \in [0^\circ ; 360^\circ]$:

$$1 + 2\cos 2x = \frac{\cos 2x}{2\sin x} - \frac{\cos x}{\sin 2x}$$ (6)

[18]

QUESTION 6

In the diagram, the graphs of $f(x) = a\sin bx$ and $g(x) = c\cos dx$ are drawn for the interval $x \in [-90^\circ ; 360^\circ]$



6.1 Determine the values of *a, b, c* and *d*. (4)

6.2 Write down the period of g. (1)

6.3 Determine the value(s) of x in the interval $x \in [-90^\circ ; 360^\circ]$, for which

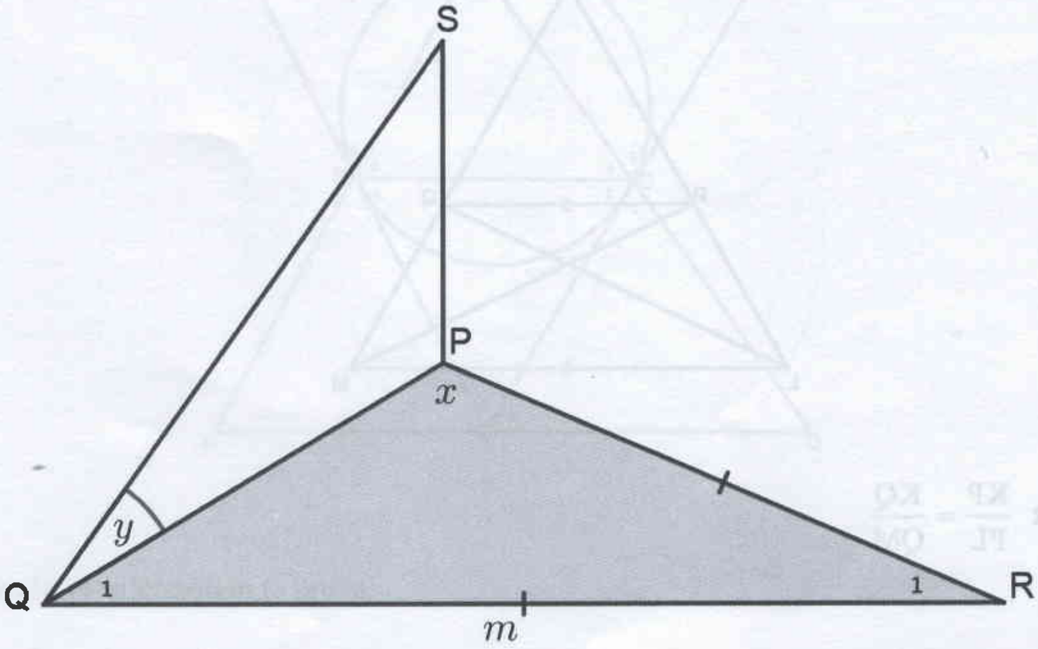
6.3.1 $f(x) \leq g(x)$ (2)

6.3.2 $f'(x) \times g'(x) > 0$ where $g(x) > 0$ (3)

[10]

QUESTION 7

In the diagram P, Q and R are three points in the same horizontal plane. PR = QR = m, $Q\hat{P}R = x$. SP is perpendicular to PQ. The angle of elevation of S from Q is y.

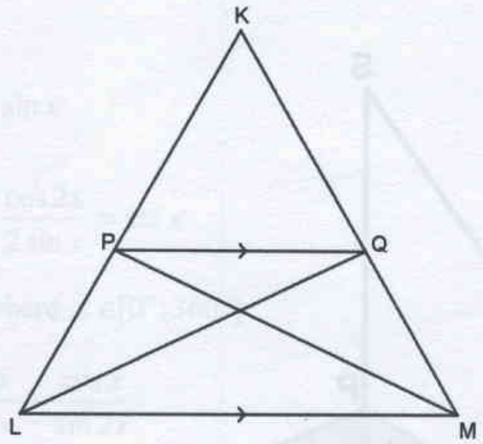


7.1	Express the area of ΔPQR in terms of x and m.	(5)
7.2	Show that PQ = $2m\cos x$	(4)
7.3	Hence, prove that SP = $2m\cos x\tan y$	(2)

[11]

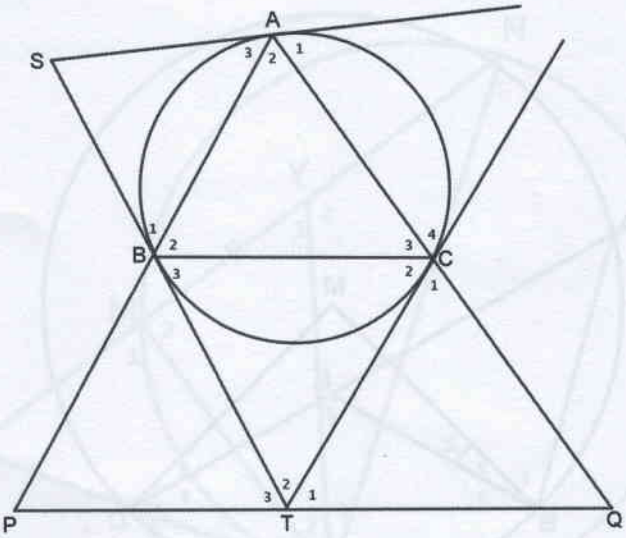
QUESTION 8

8.1 In the diagram below Δ KLM is given, with P and Q lying on KL and KM respectively such that PQ $\parallel$ LM. PM and LQ are drawn.



Prove that $\frac{KP}{PL} = \frac{KQ}{QM}$ (6)

8.2 In the diagram, SBT, SA and TC are tangents to the circle at B, A and C respectively. AB is produced to P and AC is produced to Q such that T lies on the line PQ. In ΔAPQ, $\dfrac{AB}{AP} = \dfrac{AC}{AQ}$.



Use the above information to prove:

8.2.1 $\hat{A}_2 = \hat{T}_1$ (4)

8.2.2 $\Delta ABC \mathbin{///} \Delta TCQ$ (4)

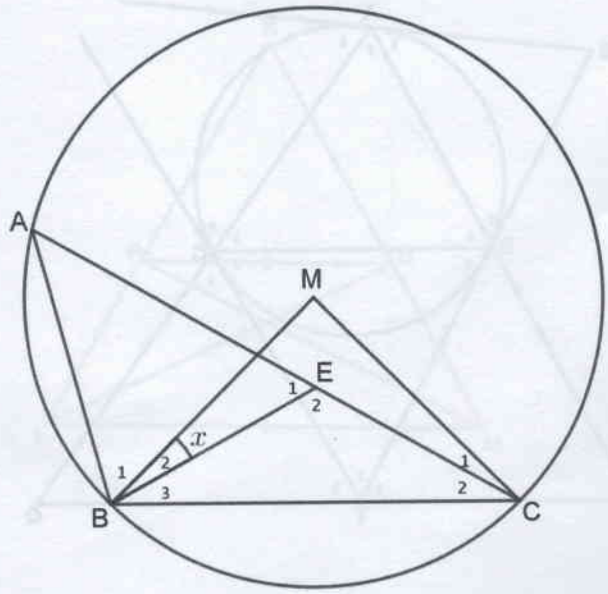
8.2.3 ABTQ is a cyclic quadrilateral. (4)

8.2.4 Prove that TQ is a tangent to circle TBC at T. (5)

[23]

QUESTION 9

In the diagram, M is the centre of the circle through A, B and C. E is on AC. AC bisects $M\hat{C}B$ and EB bisects $M\hat{B}C$. $\hat{B}_2 = x$



9.1 Determine the size of $\hat{E}_2$ in terms of x. (4)

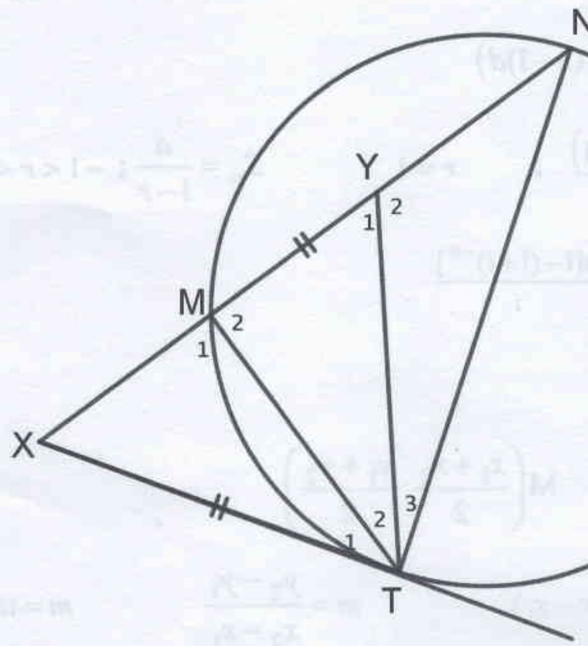
9.2 Show $B\hat{A}C = 90^\circ - 2x$ (3)

9.3 Prove that AE is a diameter of circle ABE. (5)

[12]

QUESTION 10

10.1 In the diagram XMN is a straight line and XT is a tangent to the circle.
Y is a point on XN so that XY = XT.



Prove that:

10.1.1 YT bisect $M\hat{T}N$. (5)

10.1.2 $\dfrac{XM}{XT} = \dfrac{XT}{XN}$ (6)

10.2 Given that MY = 20 mm, YN = 50 mm and XT = k mm:

10.2.1 Express XM in terms of k. (3)

10.2.2 Calculate the length of k. (4)

[18]

TOTAL MARKS: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum f.x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P}(A) = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$

$\hat{y} = a + bx$ $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$